Introduction

Queuing is a sign of modern civilization, which makes things be done in order. However, inside a granular flow, grains will be mixed in general. In some cases, the grains that enter the hopper at earlier times, however, will exit later, which is the phenomenon of reverse ordering. In this study, we quantified and investigated the reverse ordering phenomenon in a two-dimensional (2D) gravity-driven hopper flow, examined the effects of the reclining angle and the hopper angle, and analyzed the phenomenon in different scales.

Apparatus and Methods

1.Parameters and methods

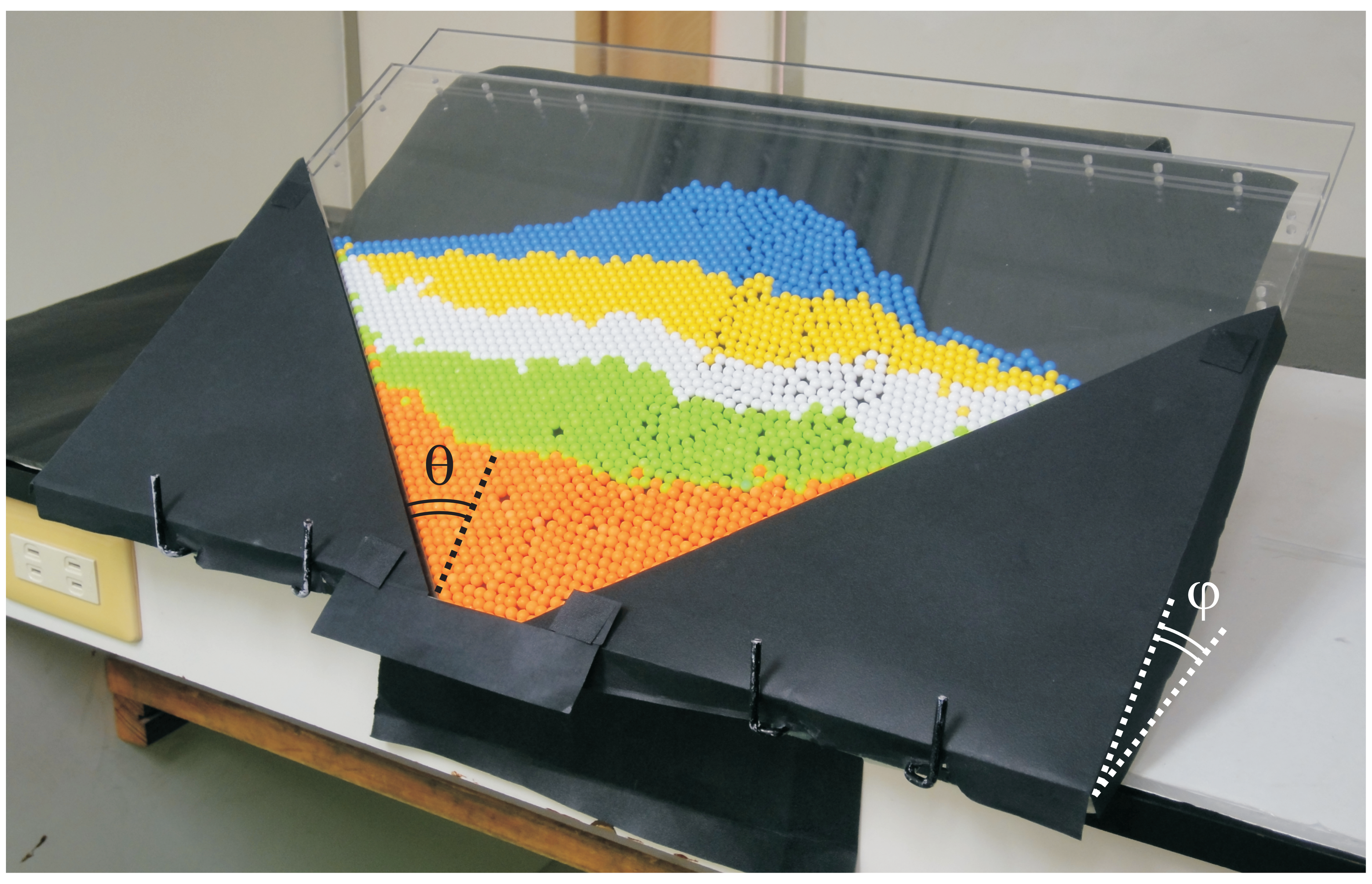


Fig.1 The hopper and experimental setup

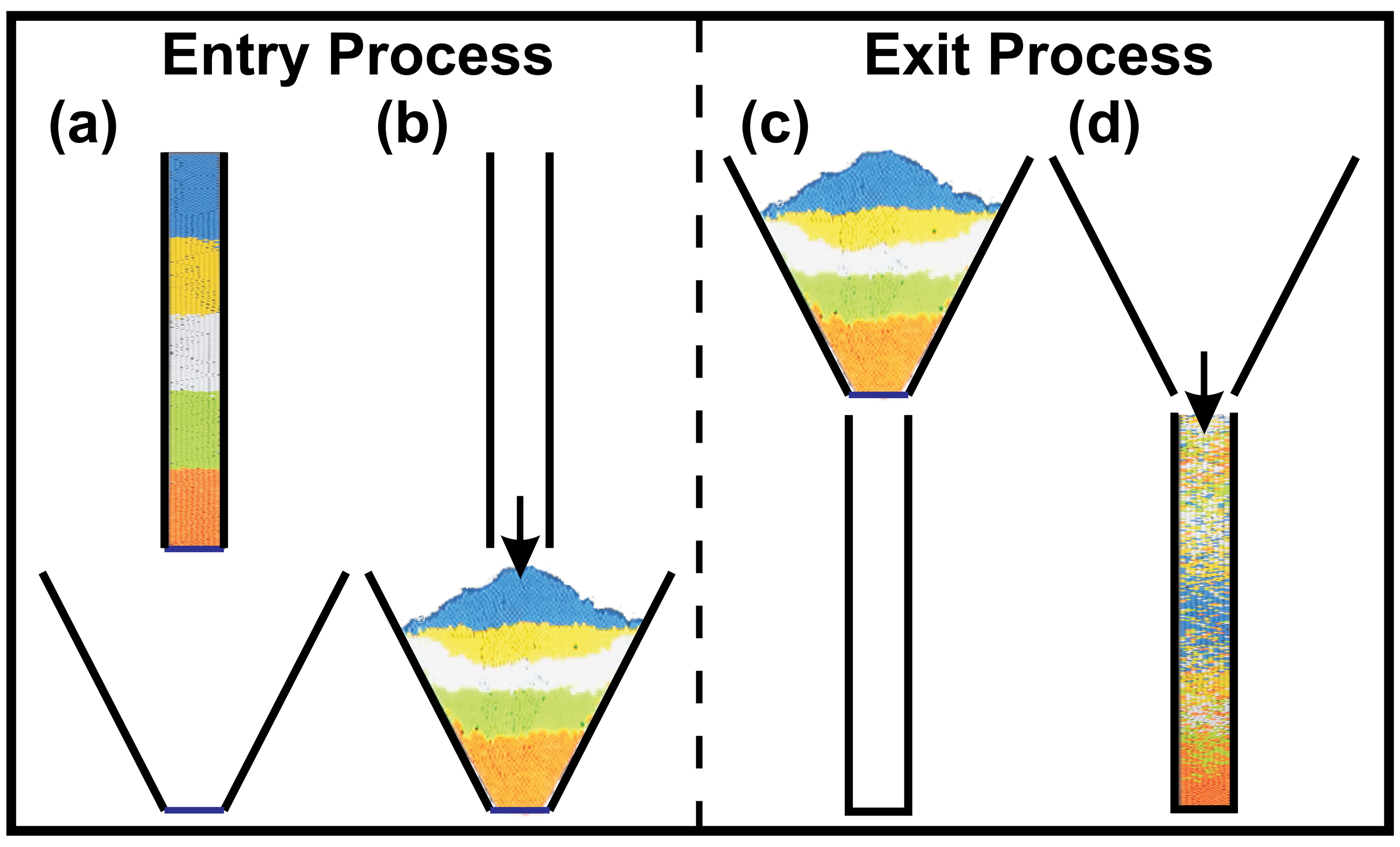
- **Physical parameters**

Plastic spheres (diameter = 6mm; BB bullets) are used in this study. We constructed 2D hoppers made of acrylic plates with different hopper angle θ (30°~60°) and adjustable reclining angle φ (15°~90°).

- **Methods**

We load grains into the hopper by a 2D channel (Fig. 2). After we open the gate, grains will flow out the hopper. The whole process is recorded by a high speed camera at 300fps.

2.The phenomenon



Grains with different colors are loaded into the 2D channel to indicate the entry order of them (Fig. 2(a)). After grains fill up the hopper(Fig. 2(b)) (Fig. 2(c)), we open the gate and make them drop into the 2D channel to indicate their exit order (Fig. 2(d)). We perform two experiments at different hopper angles (Fig. 3).

Fig. 2 Schematic of qualitative experiment

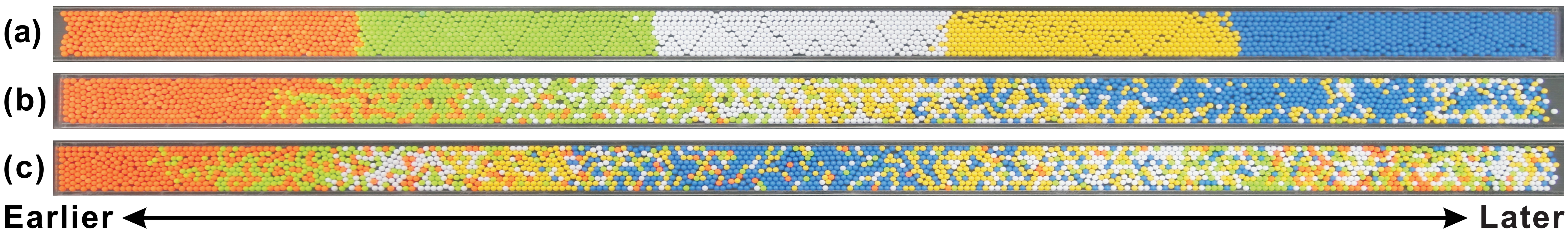


Fig. 3 (a)The entry order at θ=30°,60°, (b) the exit order at θ=30°, and (c) the exit order at θ=60°

When θ=30°, grains undergo mixing. However, grains undergo reverse ordering when θ=60°. For instance, the blue grains enter later while exit earlier.

3.Quantitative analysis

(1)We define γ_i to denote the order difference of each grain i:

$$\gamma_i = I_i - O_i$$

I_i: entry order of grain i
O_i: exit order of grain i

(2)We also define a quantity Γ to represent the degree of reverse ordering of an experiment, which ranges 0 to 1.

$$\Gamma = \frac{1}{f(N)}\sqrt{\sum_{i=1}^{N}\gamma_i^2} \qquad f(N) = \sqrt{\frac{N^3 - N}{3}}$$

N: number of grains; $f(N)$: normalizing factor

A larger Γ means the reverse ordering is more obvious.

Examples of calculating Γ :

Perfect ordering

I_i:		1	2	3	4	......	1999 2000
O_i:	-)	1	2	3	4	......	1999 2000
γ_i:		0	0	0	0	......	0 0

$$\Gamma = \frac{1}{\sqrt{(2000^3-2000)/3}}\sqrt{0^2+0^2+...+0^2+0^2} = 0$$

Perfect reverse ordering

I_i:		1	2	......	1999	2000
O_i:	-)	2000	1999	......	2	1
γ_i:		-1999	-1997	......	1997	1999

$$\Gamma = \frac{1}{\sqrt{(2000^3-2000)/3}}\sqrt{(-1999)^2+...+1999^2} = 1$$

Results and Discussion

1.Maps of grains' order

• Contour maps of entry order and exit order

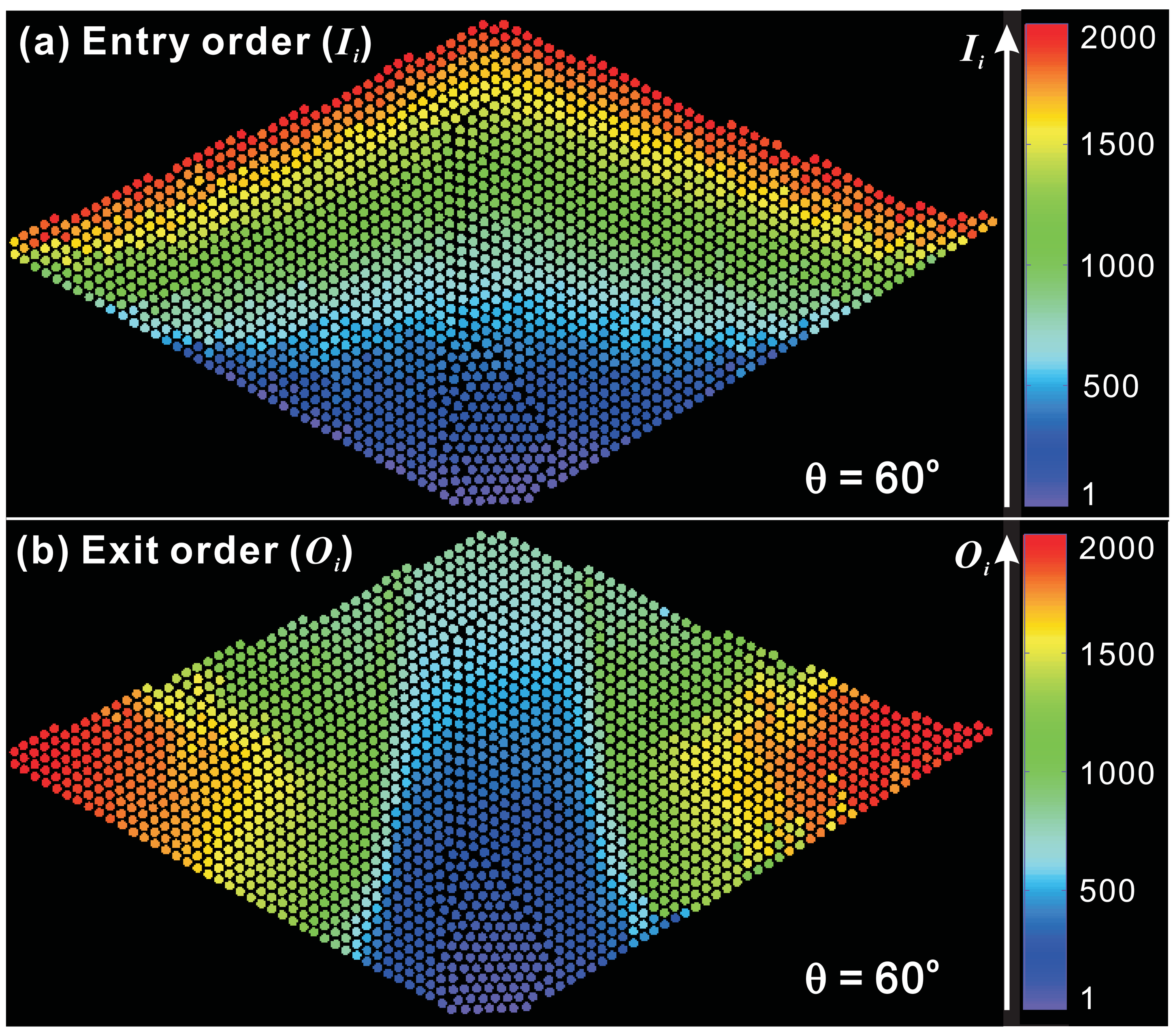


Fig. 4 Contour maps of grains' (a) entry order and (b) exit order

(1)We draw maps of grains' entry order (Fig. 4(a)) and exit order (Fig. 4(b)) according to the grains' initial locations in the hopper before they flow out.

(2)In the map of entry order, the grains with similar colors will enter the hopper together. The map looks stratified horizontally, which means grains do not mix seriously when entering the hopper (Fig. 4(a)).

(3)In the map of exit order, the grains with similar colors will exit the hopper together. The map looks stratified vertically (Fig. 4(b)), which is different from the map of entry order.

• Contour maps of order difference γ_i

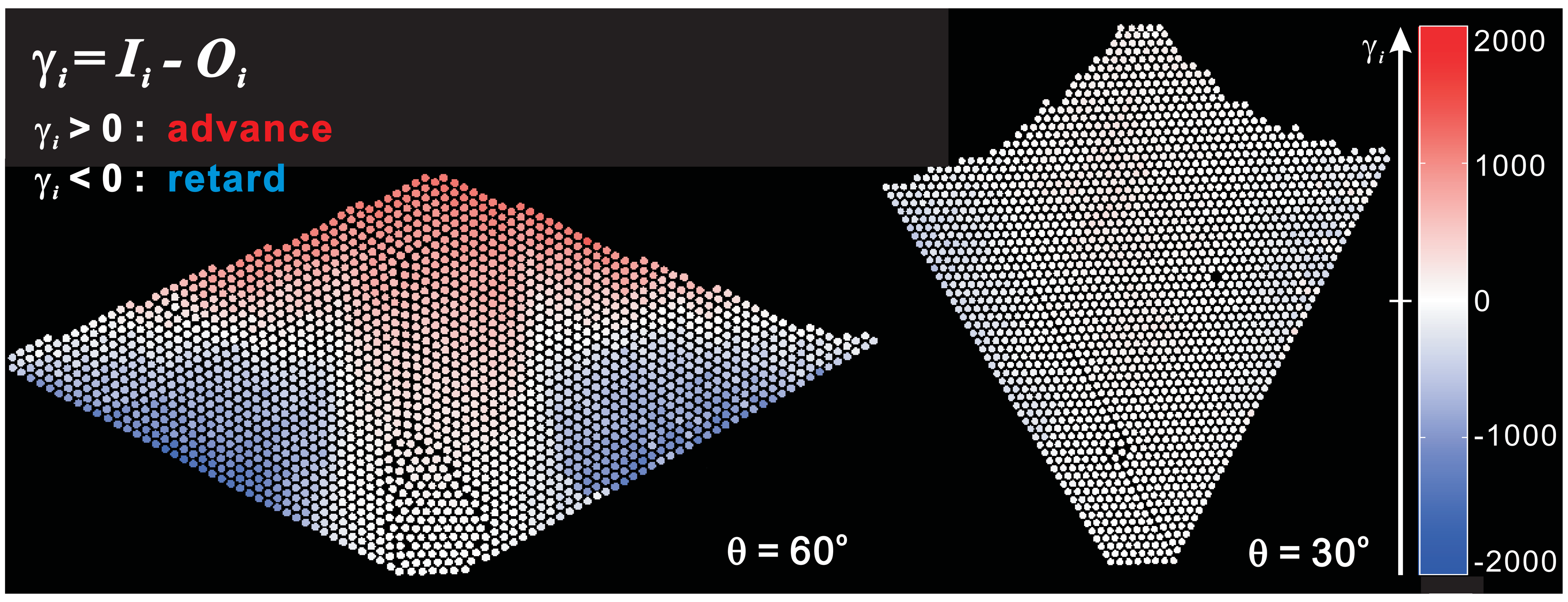


Fig. 5 Contour maps of grains' order difference γ_i at θ=60° and θ=30° (red grains: advance; blue grains: retard)

To find out the regions that grains undergo reverse ordering, we draw maps of the order difference γ_i (Fig. 5). The grains initially located at the central part and the side-wall part of the hopper undergo reverse ordering.

The colors at θ=30° are lighter than the colors at θ=60°. Hence, this indicates that the degree of reverse ordering of the former is smaller than that of the latter.

2.Effects of reclining angle and hopper angle

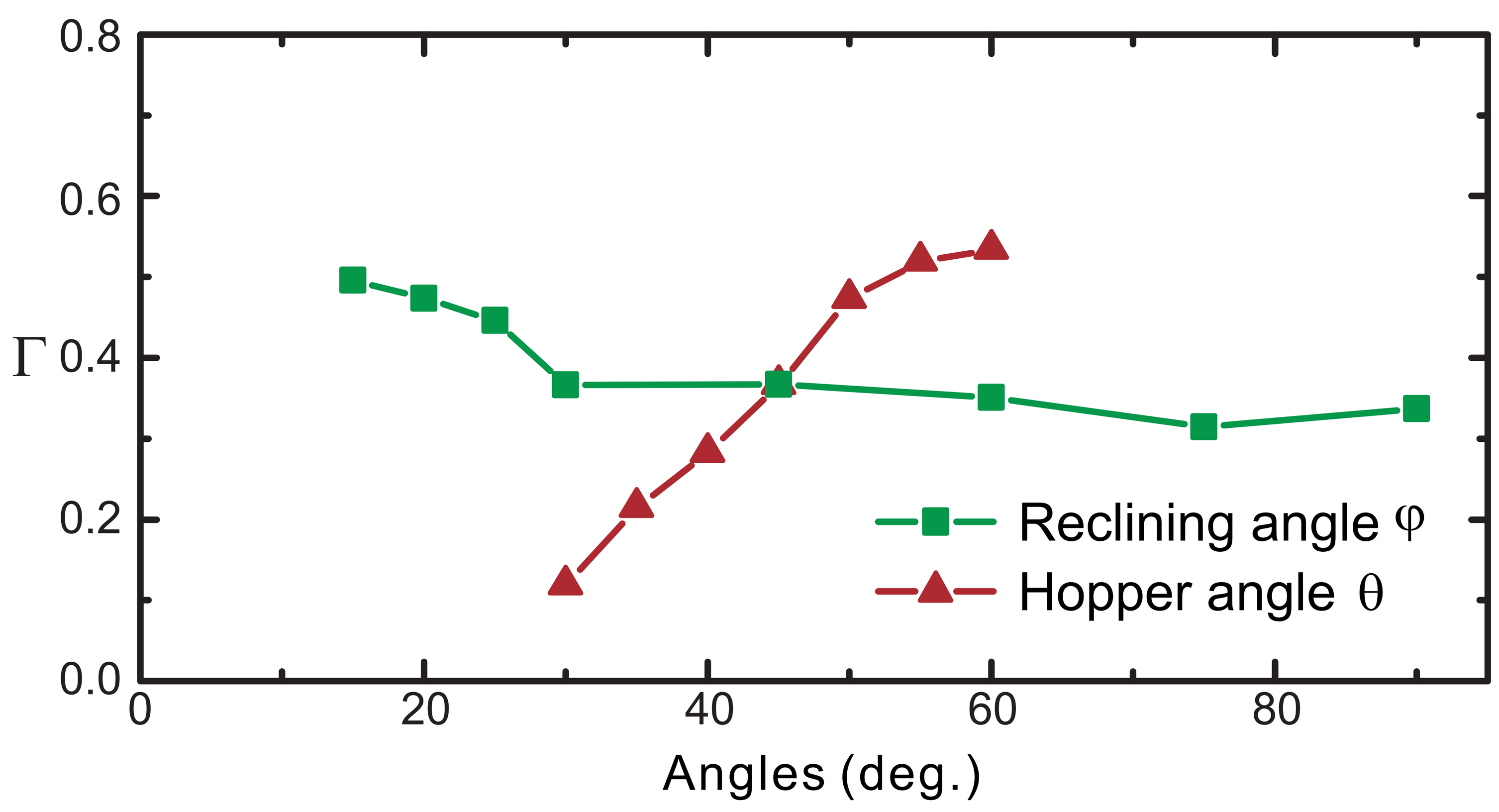


Fig. 6 Effects on the degree of reverse ordering Γ of the reclining angle and the hopper angle

(1)The degree of reverse ordering Γ increases with decreasing reclining angle φ and increasing hopper angle θ (Fig. 6).

(2)A larger reclining angle φ speeds up the flow and reduces the completion time difference between central flow and side-wall flow, which reduces the degree of reverse ordering.

(3)A larger hopper angle θ provides a larger frictional force for a grain on side wall, and this slows down side-wall flow and increases the completion time difference, which increases the degree of reverse ordering.

3.Discussion

• Central flow and side-wall flow

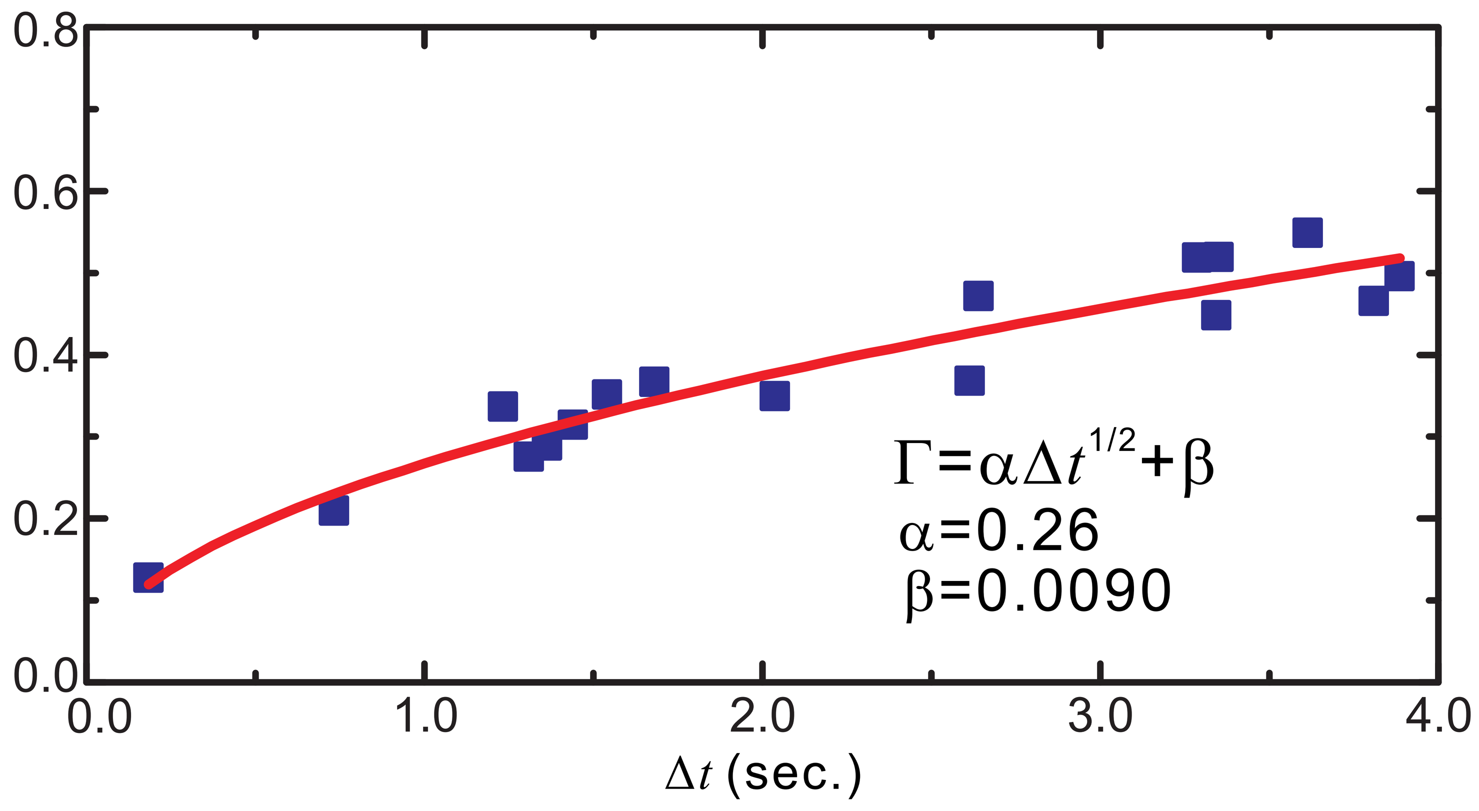


Fig. 7 Dependence of the completion time difference between central flow and side-wall flow Δt on the degree of reverse ordering Γ

(1)We think a larger degree of reverse ordering Γ results from a larger completion time difference between central flow and side-wall flow.

(2)To check our theory, we measured the completion time for central flow (t_c) and side-wall flow (t_s) and calculated their difference Δt (t_s-t_c).

(3)The degree of reverse ordering Γ is proportional to the square root of the completion time difference $\Delta t^{1/2}$ (Fig. 7).

• Reverse ordering and friction

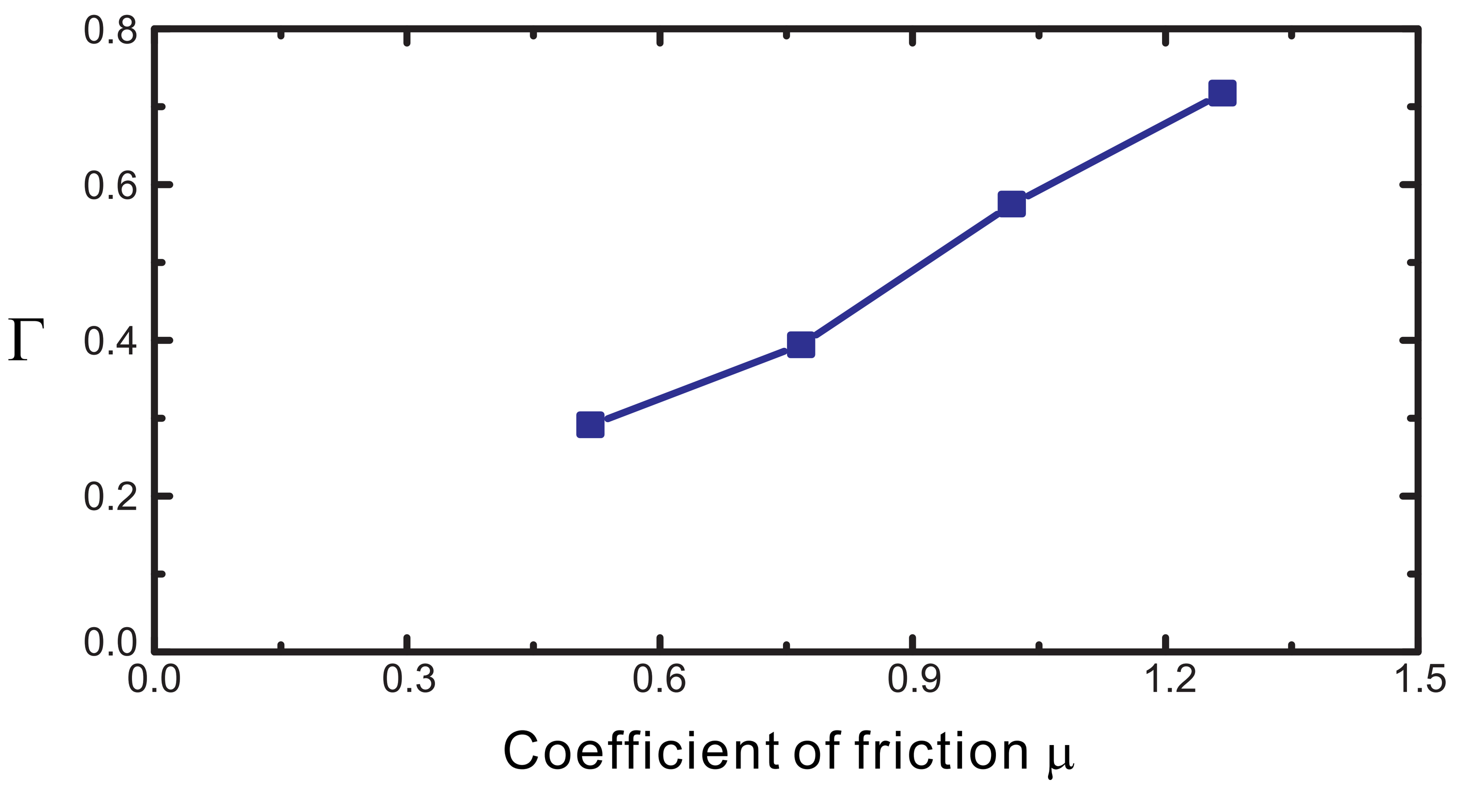


Fig. 8 Effects on the degree of reverse ordering Γ of the friction between grains and side walls

(1) We also check our theory by sticking anti-slip materials on side walls, which increase the friction between grains and side walls.

(2) The degree of reverse ordering Γ increases with increasing friction between grains and side walls (Fig. 8). A larger friction slows down side-wall flow and increases the completion time difference. This is consistent to our theory.

• Surface avalanches

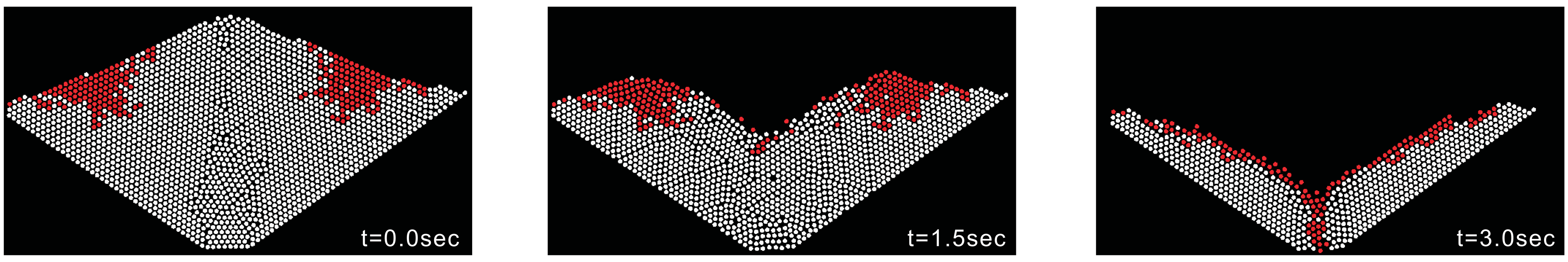


Fig. 9 Dynamical behavior of the grains that undergo surface avalanches (the red grains)

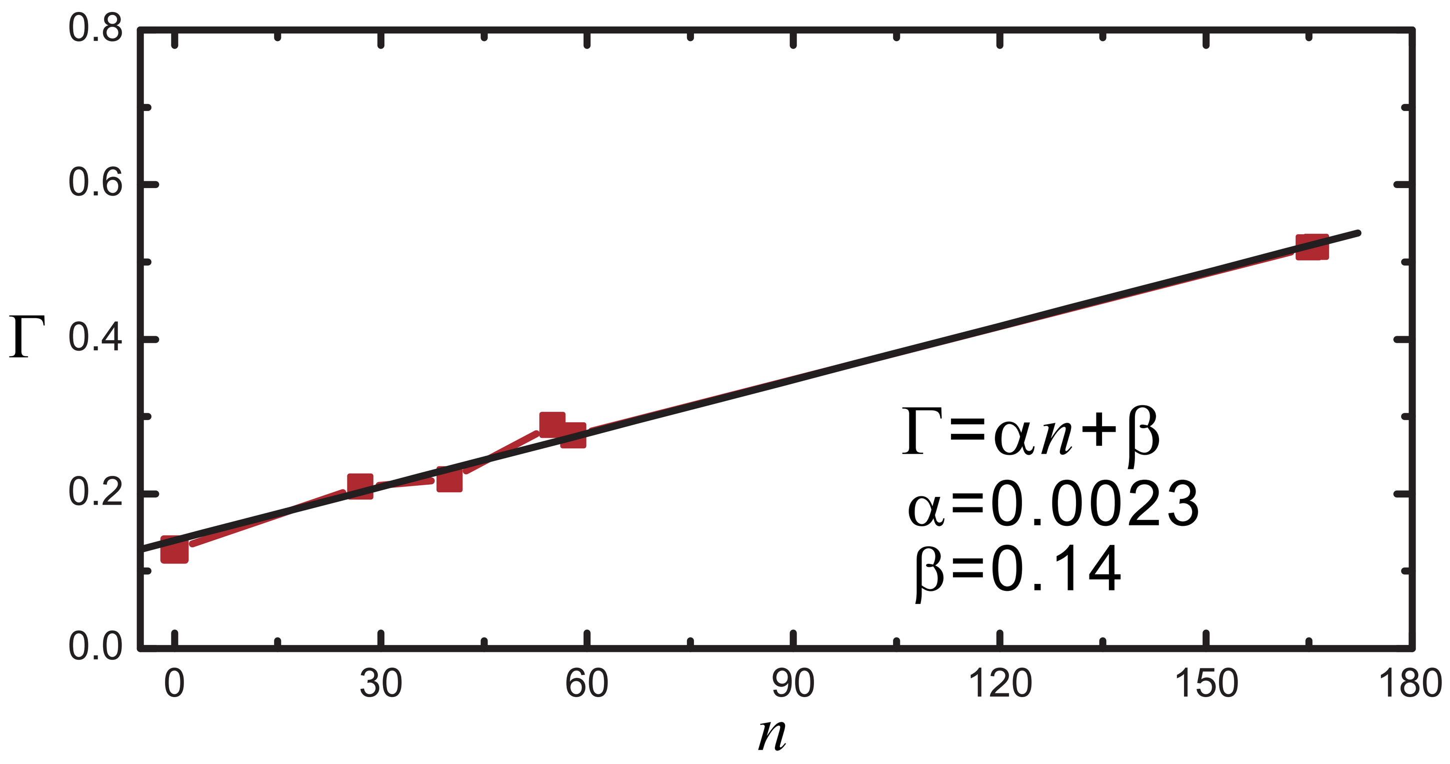


Fig. 10 Relation between the number of the grains undergoing surface avalanches n and the degree of reverse ordering Γ

(1) In some experiments, a V-shaped surface is observed, which leads to the surface avalanches (Fig. 9). We think this may influence the reverse ordering.

(2) To check our thought, we counted the grains that undergo surface avalanches n in each experiment. The degree of reverse ordering Γ increases with increasing n (Fig. 10).

Conclusions

1. A reverse ordering phenomenon is observed.
2. Grains start from the central part of the hopper will advance while those start from the side-wall part of the hopper will retard.
3. Macroscopically, the degree of reverse ordering can be quantified by Γ, the quantity we defined, which is sensitive to the hopper angle.
4. The completion time difference Δt between central flow and side-wall flow, which is a mesoscopic property, leads to the reverse ordering of grains, and Γ is proportional to $\Delta t^{1/2}$.
5. Microscopically, grains undergo avalanches at the surface of the flow, which also influences the reverse ordering.

Applications

These results may be useful for special hopper design in agricultural and pharmaceutical industries. The reverse ordering may influence the dropping order of grains while packing or transporting, making them spoil and stranded, even affect the freshness of the items such as millet, medicines, and ores.

References

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Photos taken by authors.

Reverse Ordering in Dynamical Two-Dimensional Hopper Flow

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